Supplementary Information for "Transition and formation of the torque pattern of undulatory locomotion in resistive force dominated media"

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Torque calculation based on elongated body theory

The same as in the main paper, the lateral displacement y of the swimmer in its body frame can be described as,

$$y = h(x,t) = A\sin[2\pi(x/\lambda + t/T_p)], \tag{1}$$

where A is the amplitude, λ is the wavelength, and T_p is the period. The velocity of the bending wave is $v = \lambda/T_p$. According to the elongated body theory proposed by Lighthill [1], the lateral force on the swimmer per unit length exerted by the water is:

$$L(x,t) = -\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\{\rho A(x)V(x,t)\}, \tag{2}$$

where, U is the swimming speed, $\rho A(x)$ is the "added mass", and $V(x,t) = \frac{\partial h}{\partial t} + U\frac{\partial h}{\partial x}$ is the velocity of the body at x relative to the fluid. We set U to a typical value of $4/5v$; .

The prescribed motion in Eq.1 does not satisfy the rigid body dynamics, therefore the lateral and rotational movements (recoil motion) of the swimmer have to be added to the prescribed motion:

$$h_a(x,t) = h(x,t) + F(t) + xG(t), \tag{3}$$

where $F(t)$ is the transverse recoil and $G(t)$ is the angular recoil.

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$F(t)$ and $G(t)$ are solved from

$$\begin{aligned}\int_0^l \rho S(x)\frac{\partial^2 h_a}{\partial t^2}dx &= \int_0^l L_a(x,t)dx,\\ \int_0^l x\rho S(x)\frac{\partial^2 h_a}{\partial t^2}dx &= \int_0^l xL_a(x,t)dx\end{aligned} \tag{4}$$

where ρ and $S(x)$ are the density and the area of cross-section of the swimmer, respectively. We assume the density of the swimmer is the same as the water and the $S(x)$ is a constant along the body. We further assume the body of the swimmer is a cylinder such that $S(x) = A(x) = \text{const}$.

The analytical forms of $F(t)$ and $G(t)$ are listed as follows:

$$\begin{aligned}F(t) &= -\frac{D\lambda T_p^2}{8\pi^3 l}[-\frac{3}{2\pi l}(\lambda + T_pU), \frac{3}{2\pi l}(\lambda - T_pU), \frac{3\lambda T_pU}{2\pi^2 l^2} - 2, -\frac{3\lambda T_pU}{2\pi^2 l^2} - 1]X^T,\\ G(t) &= -\frac{3D\lambda T_p^2}{8\pi^3 l^3}[\lambda/\pi, -\lambda/\pi, l, l]X^T,\\ X &= \{\sin(2\pi t/T_p), \sin[2\pi(l/\lambda + t/T_p)], \cos[2\pi(t/T_p)], \cos 2\pi[(l/\lambda + t/T_p)]\},\end{aligned} \tag{5}$$

where $D = -4A\pi^2(\frac{2}{T_p^2} + \frac{2U}{T_p\lambda} + \frac{U^2}{\lambda^2})$.

With the actual lateral movement $h_a(x,t)$ and corresponding lateral force L_a, we can calculate the torque at a point x by integrating the torque contribution from either the anterior or the posterior side of the body:

$$T(x,t) = \int_0^x (s-x)\left\{L_a(s,t) - \rho S\frac{\partial h_a^2}{\partial t^2}\right\}\mathrm{d}s. \tag{6}$$

The term $-\rho S\frac{\partial h_a^2}{\partial t^2}$ in Eq. 6 represents the inertial force of the body. Finally, we obtain the torque $T(x,t)$ as follows:

$$T(x,t) = -\rho S\{[c_1, c_2, c_3, c_4]X^T - \frac{D\lambda^2}{4\pi^2}\sin 2\pi(t/T_p) + \frac{D\lambda^2}{4\pi^2}\sin 2\pi(x/\lambda + t/T_p) - \frac{D\lambda x}{2\pi}\cos 2\pi(t/T_p)\} \tag{7}$$

Where

$$\begin{aligned}c_1 &= \frac{D\lambda^2}{4\pi^2}(\frac{3x^2}{l^2} - \frac{2x^3}{l^3}),\\ c_2 &= -c_1,\\ c_3 &= \frac{D\lambda x}{2\pi}(\frac{2x}{l} - \frac{x^2}{l^2}),\\ c_4 &= \frac{D\lambda x^2}{2\pi l}(1 - \frac{x}{l}).\end{aligned} \tag{8}$$

References

[1] MJ Lighthill. Note on the swimming of slender fish. *Journal of Fluid Mechanics*, 9(02):305–317, 1960.

Caption for SI Video S1

Movement and force distribution of the force on the swimmer. Force per unit length and velocity are scaled by 0.1.

Caption for SI Video S2

The torque pattern of undulatory locomotion from viscous drag forces as the wave number ξ increases.

Caption for SI Video S3

The torque pattern of undulatory locomotion from frictional forces as the wave number ξ increases.

Caption for SI Video S4

The torque pattern of undulatory locomotion from granular forces as the wave number ξ increases.

Caption for SI Video S5

The torque pattern predicted by elongated body theory for swimming at high Re as the wave number ξ increases.

Caption for SI Video S6

The torque pattern as a function of the wave number ξ. A large amplitude $A = 12.57$ and viscous force laws are used.

Caption for SI Video S7

The torque pattern as a function of the wave number ξ. An increasing amplitude from the head to tail $A = 7.54(1 - s)$ and viscous force laws are used.

Caption for SI Video S8

The torque pattern as a function of the wave number ξ. Head drag is included and viscous force laws are used.

Caption for SI Video S9

The torque pattern as a function of the amplitude A. Viscous force laws are used.

Caption for SI Video S10

The torque pattern as a function of the wave number ξ, computed using Slender Body Theory.

Caption for SI Video S11

The torque pattern as a function of the wave number ξ. Contribution from elastic body forces are considered.

Caption for SI Video S12

The torque pattern as a function of the wave number ξ. Contribution from viscous body forces are considered.