

Supplementary Information for “3D computational models
explain muscle activation patterns and energetic functions of
internal structures in fish swimming”

Torque and power patterns from pure resistive forces and reactive forces

To compute the torque and power patterns associated with pure resistive forces and reactive forces, we consider simplified cases which can be solved analytically. We assume that the body is uniform and the curvature wave is prescribed by $\kappa(s, t) = A \sin(2\pi s + 2\pi t)$, where $A \ll 1$, $0 \leq s \leq 1$, and $0 \leq t \leq 1$. It is easy to obtain that the lateral motions resulted from resistive forces $\mathbf{F}_d = -C_d \mathbf{v}$ or reactive forces $\mathbf{F}_i = -m\mathbf{a}$ can be both approximated as :

$$y(x, t) = -A \left[\sin(2\pi t + 2\pi x) + \frac{6 \left(x - \frac{1}{2}\right) \cos(2\pi t)}{\pi} \right] \quad (\text{S1})$$

The torque associated can be computed as

$$T(s, t) \approx - \int_s^1 x F_d = \int_s^1 x C_d v_y dx = C_d [\cos(2\pi t) - \cos(2\pi x - 2\pi t) + 2\pi x(1 - 3x + 2x^2) \sin(2\pi t)] / (2\pi), \quad (\text{S2})$$

and

$$T(s, t) \approx - \int_s^1 x F_i = - \int_s^1 x m a_y dx = m [-\sin(2\pi t) - \sin(2\pi x - 2\pi t) + 2\pi x(1 - 3x + 2x^2) \cos(2\pi t)], \quad (\text{S3})$$

,respectively. In the above analysis, m is a constant which represents the added mass per unit length.